LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – STATISTICS

FIFTH SEMESTER – November 2009

ST 5501 - TESTING OF HYPOTHESIS

Date & Time: 5/11/2009 / 9:00 - 12:00 Dept. No. Max. : 100 Marks

PART - A

Answer *ALL* the questions (10 x 2 = 20 marks)

1. A statistician inspects a production lot consisting of 100 units. He decides to reject the hypothesis $H:\theta = 0.1$ if the number of defective units in a sample of 10 units drawn from the lot exceeds 2. Compute the probability of committing an error of Type-I assuming the units are drawn one by one with replacement.

2. What are randomized tests? Mention their significance.

3. Define: Uniformly Most Powerful tests.

4. Show that Poisson density functions are members of one parameter exponential family.

5. Mention the important difference between usual statistical tests and SPRTs
6. Define Likelihood ratio criterion.

7. When do you recommend the use of paired t-test?

8. Give any two tests of significance based on chi-square distribution.

9. Define: Mann-Whitney U-statistic.

10. What are the assumptions made while using Non-parametric methods?

PART - B

Answer any FIVE questions (5 x 8 = 40 marks)

11. Derive the Most Powerful test for testing $H: N = 8$ against the hypothesis $K: N = 12$ based on a sample of size 2 drawn from a population with probability density function

$$P_N(x) = \begin{cases} \frac{1}{N}, x = 1,2,\dots,N \\ 0, \text{otherwise} \end{cases},$$

N being a positive integer, assuming $\alpha = 0.05$.

12. Show that the family of Uniform densities $U(0,\theta), \theta > 0$ has Monotone Likelihood Ratio property.

13. Derive the UMPT of level 0.05 for testing $H:\lambda \le 0.01$ against $K:\lambda > 0.01$ based on a sample of size 10 drawn from Poisson population with mean λ.

14. Write a descriptive note on Sequential Probability Ratio Test.

15. Derive the likelihood ratio test for testing $H:\theta=\theta_0$ against $K:\theta \ne \theta_0$ in $N(\theta,\sigma^2)$ assuming that the sample size is n (σ^2 unknown).

16. Derive the large sample confidence interval for θ, the mean of exponential distribution.

17. Explain sign test for randomness.

18. Assume samples of sizes 10 and 9 are drawn from two populations with distribution functions F and G, whose values are given below.

    4.3, 5.9, 4.9, 3.1, 5.3, 6.4, 6.2, 3.8, 7.5, 5.8
    5.5, 7.9, 6.8, 9.0, 5.6, 6.3, 8.5, 4.6, 7.1

    Test the hypothesis $H:F(z)=G(z)$ for all z using an appropriate Non-parametric test.

PART - C

Answer any TWO questions **(2 x 20 = 40 marks)**

19. (a) State and prove Neyman Pearson lemma.

    (b) Let X be an observation drawn from exponential distribution with mean θ. Suggest any two tests having size 0.05 and compare their powers under alternative for testing $H:\theta=1$ against $K:\theta=2$.

20. Derive the Uniformly most powerful test of level 0.05 for testing $H:\theta \le 3$ against $K:\theta > 3$ based on a sample of size 10 drawn from a population with probability density function $p_\theta(x)=\begin{cases} e^{-(x-\theta)}, x>\theta \\ 0, \text{otherwise}\end{cases}$ and also obtain its power function.

21. Derive the likelihood ratio test for testing the equality of two normal means assuming that the population variances are equal.

22. (a) Explain the usage of Kolmogrov-Smirnov two sample test.

    (b) Find he sequential probability ratio test for testing $H:\theta=75$ against $K:\theta=78$ in $N(\theta, 100)$ assuming α and β are approximately equal to 0.10.

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